HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS – ADVANCED

2014 HSC

HALF YEARLY EXAMINATION

ASSESSMENT TASK 2

Examiners ~ G Huxley, S Hackett, S Faulds, D. Crancher, P. Biczo

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes.
- Working Time – 2 hours.
- Attempt **all** questions.
- **All** necessary working should be shown in every question.
- This paper contains six (6) multiple choice questions, and 6 extended response questions, worth 11 marks each. Total = 72 marks.
- Multiple choice is to be answered on the sheet provided. The extended response questions are to be answered in the booklets provided.
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators may be used. Approved templates are optional.
- **Each question is to be started in a new booklet.**
- This examination paper must **NOT** be removed from the examination room.
- Write your student number/name on each answer sheet.

STUDENT NAME:______________________________

TEACHER: ______________________________

PART A **Multiple Choice (Complete on the answer sheet provided)**

QUESTION 1

Given $y=(5-3x^2)^6$, $\frac{dy}{dx}=?$

A. $-6x(5-3x^2)^5$ B. $-36x(5-3x^2)^5$

C. $-6(5-3x^2)^5$ D. $-36(5-3x^2)^5$

QUESTION 2

What is the gradient of the normal to the function $y=7x-6$ at the point where $x = a$?

A. $7a$ B. $-\frac{a}{7}$ C. $-\frac{1}{7}$ D. $\frac{1}{7}$

QUESTION 3

The quadratic equation in x with roots $2+\sqrt{3}$ and $2-\sqrt{3}$ is

A. $x^2-4x+1=0$ B. $x^2+4x+1=0$

C. $x^2-4x-1=0$ D. $x^2+4x-1=0$

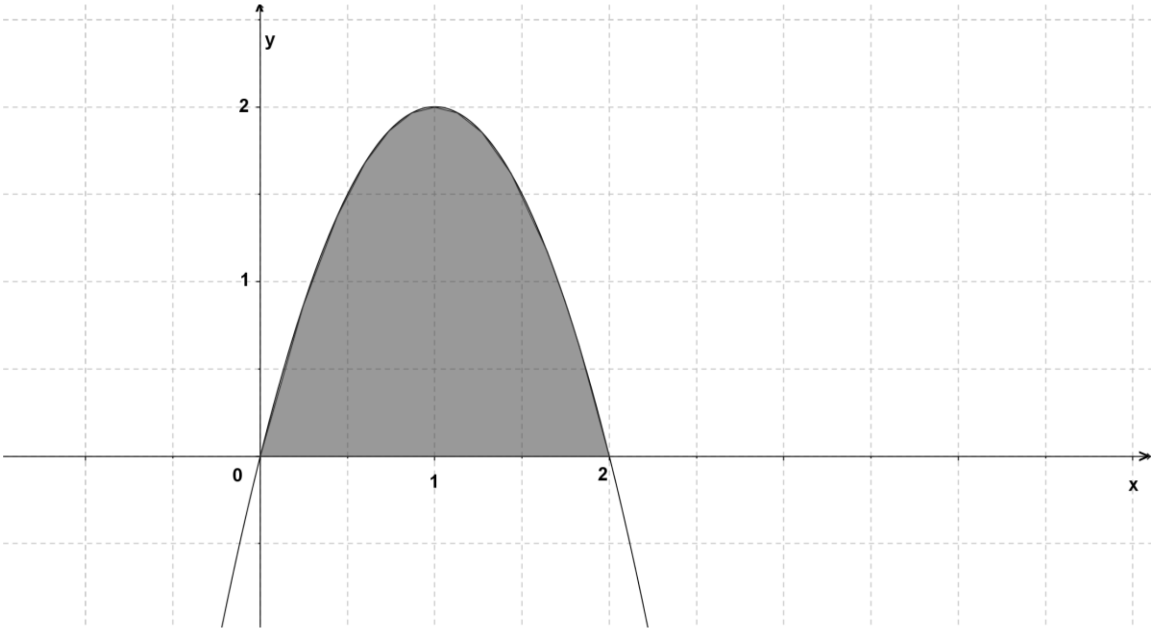
QUESTION 4

A point P(x, y) moves so that its distance from the point (0, 2) is 2 units. The equation of the locus of point P is:

A. $x^2 = 8y$ B. $x^2 + (y-2)^2 = 4$

C. $(x-2)^2 + y^2 = 4$ D. $x^2 = -8y$

QUESTION 5



Which of the following could be used to calculate the shaded area above?

A. $\int_0^2 (x-2)^2\, dx$

B. $\int_0^2 2x\,(x-2)\,dx$

C. $\int_0^2 -2x\,(x-2)\,dx$

D. $\int_0^2 -x\,(x-2)\,dx$

QUESTION 6

The fourth term of an arithmetic series is 27 and the seventh term is 12.
What is the common difference?

A: 42 B: 13 C. 5 D. –5

PART B:

QUESTION 7 11 marks *Start a SEPARATE booklet.* **Marks**

Consider the function $f(x)=\dfrac{x-1}{x^2}$.

(i) Show that $f'(x)=\dfrac{2-x}{x^3}$. **2**

(ii) Find the coordinates of the stationary point on $y=f(x)$ and determine its nature. **2**

(iii) Find the coordinates of P, the only point where $y=f(x)$ meets the x-axis **1**

(iv) Calculate $\lim_{x\to\infty} f(x)$ **2**

(v) Show that the equation of the tangent at P is given by the equation $y=x-1$. **1**

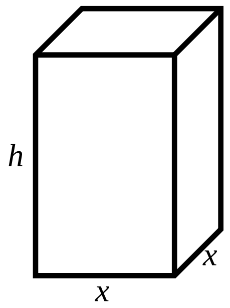
(vi) Find the coordinates of the other point where this tangent meets the curve. **3**

QUESTION 8 BEGINS ON THE NEXT PAGE

QUESTION 8 **11 marks** ***Start a SEPARATE booklet.*** **Marks**

(a) The curve $y = ax^3 + bx - 3$ has a local minimum turning point at $(-1, -4)$. Find a and b. **3**

(b) A box in the shape of a square prism is open at one of the square ends.
It has a volume of $32\,\text{cm}^3$.
The square base has length x cm and the box is h cm high.

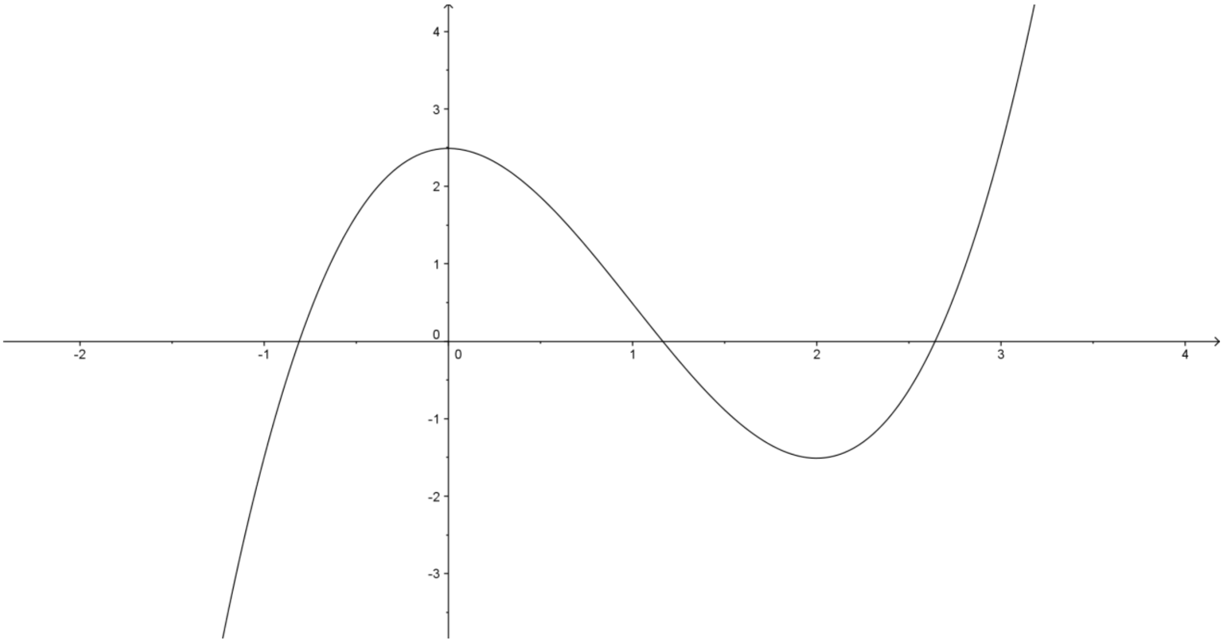


(i) Show that the surface area (A) of the box is given by: $A = x^2 + \dfrac{128}{x}$. **1**

(ii) Find the dimensions of the box that has the least surface area. **3**

QUESTION 8 continued **Marks**

(c)



The diagram shows the graph of a function $y = f(x)$.

(i) For which values of x is the derivative, $f'(x)$ positive? **1**

(ii) What happens to $f'(x)$ for large values of x? **1**

(iii) Sketch the graph $y = f'(x)$. **2**

QUESTION 9 BEGINS ON THE NEXT PAGE

QUESTION 9 11 marks *Start a SEPARATE booklet.* **Marks**

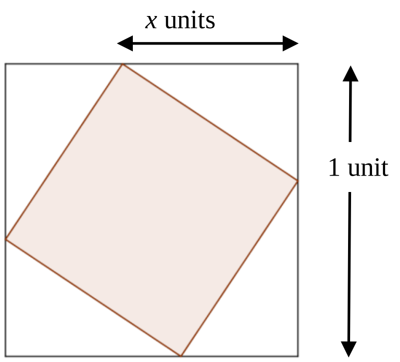
(a) If α and β are the two roots of $2x^2 - 3x + 4 = 0$

find the value of $\alpha + \beta + \alpha\beta$ **1**

(b) Solve for x: $x^2 + 1 + \dfrac{25}{x^2 + 1} = 10.$

(Hint: You may choose to use the substitution $X = x^2 + 1$) **2**

(c)



The diagram shows a square inscribed in a square of side length 1 unit. The four triangles shown are congruent, with one side of the triangle x units as shown.

(i) Show that the area of the shaded square is given by

$A = 2x^2 - 2x + 1$ **1**

(ii) **Without using Calculus**, find the minimum value of A. **2**

(d) Find the values of a and b if

$20x - 17 \equiv a(x - 4) - b(5x + 1).$ **2**

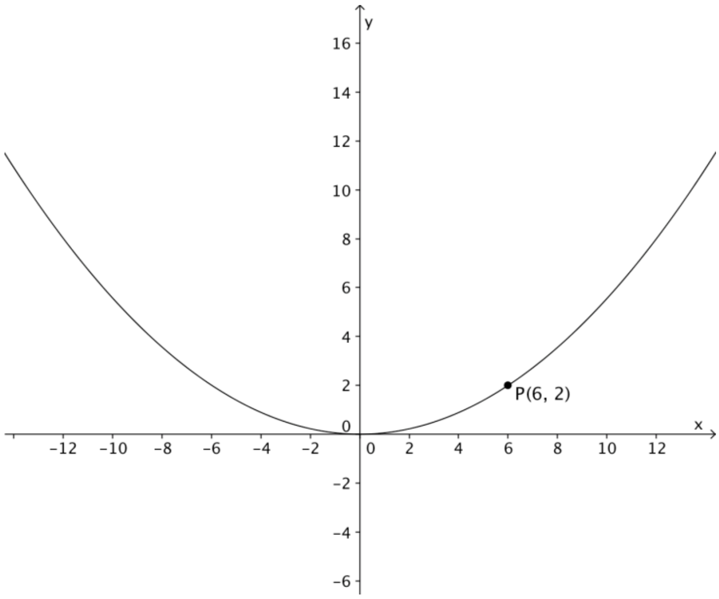
(e) Find the values of k for which

$x^2 + 2kx + k + 20 = 0$ has real roots. **3**

QUESTION 10 BEGINS ON THE NEXT PAGE

QUESTION 10 11 marks *Start a SEPARATE booklet.* **Marks**

(a) Find the co-ordinates of the focus of the parabola shown below. **2**



(b) For the parabola $(x-3)^2 = -12y$:

(i) state the co-ordinates of the focus. **1**

(ii) state the equation of the directrix **1**

(iii) sketch the parabola **1**

(iv) show that a general point $Q(x, y)$ that is equidistant from the focus and the directrix, also lies on the given parabola. **2**

(c) (i) Show that $5x + 2y = 6$ is a focal chord of the parabola $x^2 = 12y$. **2**

(ii) If $ax + by = 6$ is to be a focal chord of the parabola $x^2 = 12y$, comment upon the range of possible values for a and b. **2**

QUESTION 11 BEGINS ON THE NEXT PAGE

QUESTION 11 11 marks *Start a SEPARATE booklet.* **Marks**

(a) (i) Show that $y=x^3+x$ is an odd function. **1**

(ii) Hence or otherwise, evaluate: $\int_{-2}^{2} x^3+x \ dx$ **1**

(b) (i) Draw the graph of the function $y=\sqrt{4-x^2}$ **1**

(ii) Hence or otherwise, evaluate: $\int_{0}^{2} \sqrt{4-x^2}\, dx$ **1**

(c) (i) Show that the points of intersection of the curves

$y=2x-2$ and $y=8x-7-x^2$

are $(1,0)$ and $(5,8)$ **2**

(ii) Sketch the curves on the same diagram and shade the region enclosed between them. Mark the intercepts on the axes, and the points of intersection of the curves. **2**

(iii) By evaluating an appropriate integral, calculate the area of the shaded region in (ii) **3**

QUESTION 12 BEGINS ON THE NEXT PAGE

QUESTION 12 11 marks *Start a SEPARATE booklet.* **Marks**

a) Evaluate $\sum_{k=1}^{4} (-1)^k k^2$ **1**

b) Heather decides to swim every day to improve her fitness level.
On the first day she swims 750 metres, and on each day after that she swims 100 metres more than the previous day.
That is, she swims 850 metres on the second day, 950 metres on the third day and so on.

(i) Write down a formula for the distance she swims on the *n*th day. **1**

(ii) How far does she swim on the 10^{th} day? **1**

(iii) What is the total distance she swims in the first 10 days? **1**

(iv) After how many days does the total distance she has swum equal the width of the English Channel, a distance of 34 kilometres? **2**

c) Joe borrows \$200 000 which is to be repaid in equal monthly instalments.
The interest rate is 7.2% per annum reducible, calculated monthly.
It can be shown that the amount, $\$A_n$, owing after the *n*th repayment is given by the formula:

$$A_n = 200\,000r^n - M(1 + r + r^2 + \cdots + r^{n-1}),$$

where $r = 1.006$ and $\$M$ is the monthly repayment. (Do NOT show this.)

(i) The minimum monthly repayment is the amount required to repay the loan in 300 instalments.
Find the minimum monthly repayment. **3**

(ii) Joe decides to make repayments of \$2800 each month from the start of the loan.
How many months will it take for Joe to repay the loan? **2**

END OF EXAMINATION

STUDENT NAME:…………………….. ………………

HURLSTONE AGRICULTURAL HIGH SCHOOL

2014 HSC
Half Yearly Examination

ADVANCED MATHEMATICS

PART A Answers

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax},\ \ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

QUESTION 7 – Year 12 Mathematics Half Yearly 2014

Outcomes Addressed in this Question:

P7 determines the derivative of a function through routine application of the rules of differentiation.

	SAMPLE SOLUTION	
	a) (i) $f'(x)=\dfrac{x^2.1-(x-1).2x}{(x^2)^2}$ $=\dfrac{x^2-2x^2+2x}{x^4}$ $=\dfrac{-x^2+2x}{x^4}$ $=\dfrac{-x+2}{x^3}$ $=\dfrac{2-x}{x^3}$	**2 marks** – correct answer clearly showing how the answer was reached **1 mark** – substantial progress towards correct answer *This part was generally done well when the quotient rule was used; however a number of students opted to use the product rule with negative indices. This method was not as successful.* *In a "show that" question, it is essential that enough steps are shown.*
	a) (ii) $Stationary\ \ pt: f'(x)=0 \qquad \therefore x=2\rightarrow y=\dfrac{1}{4}$ x: 1, 2, 3 $f'(x)$: 1, 0, $-\dfrac{1}{27}$ $\therefore Max\ at\ \left(2,\dfrac{1}{4}\right)$	**2 marks** – correct stationary point with nature correctly determined **1 marks** – substantial progress towards correct solution *Many students chose to use f"(x) to determine the nature of the stationary point, but did not find the second derivative or evaluate it correctly.* *A number of students did not find the y coordinate of the point.*
	a) (iii) $When\ \ f(x)=0,\ x=1\ \therefore P(1,0)$	**1 mark –** correct answer
	a) (iv) $\lim\limits_{x\to\infty}\dfrac{x-1}{x^2}=\lim\limits_{x\to\infty}\dfrac{\frac{x}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}}$ $=\lim\limits_{x\to\infty}\dfrac{\frac{1}{x}-\frac{1}{x^2}}{1}$ $=\dfrac{0-0}{1} \qquad (as\ x\rightarrow\infty, \dfrac{1}{x}\rightarrow 0\ and\ \dfrac{1}{x^2}\rightarrow 0)$ $=0$	**2 marks** – correct answer clearly showing how the answer was reached **1 mark** – substantial progress towards correct answer *Many students correctly found the limit as zero, but did not provide an explanation of how they arrived at their answer. Some justification was necessary to be awarded the two marks for this question.*
	a) (v) $When\quad x=1,\quad f'(x)=\dfrac{1}{1}=1$ $Eqn:\quad y-0=1(x-1)$ $y=x-1$	**1 mark** – correct answer clearly showing how the given equation was reached *Most students did this part well. In a "show that" question, steps MUST be shown.*
	a) (vi) $y=x-1 \qquad -(1)$ $y=\dfrac{x-1}{x^2} \qquad -(2)$ $Solving: \quad x-1=\dfrac{x-1}{x^2}$ $x^2(x-1)=x-1$ $x^2(x-1)-(x-1)=0$ $(x-1)(x^2-1)=0$ $(x-1)(x-1)(x+1)=0$ $\therefore x=1,1,-1$ $Since\ P(1,0), the\ other\ pt\ is\ (-1,-2)$	**3 marks –** correct point obtained with a correct method of solving the equations **2 marks** – correct point with an error in the method of solution **1 mark** – substantial progress towards correct answer *Students used a wide range of methods to solve these equations – some of which were very complicated. Many of the methods lost a solution, or resulted in a contradiction where x=1 was given as a solution but they had divided by x-1 to get that solution. Some students neglected to provide the y coordinate.*

QUESTION 8 – Year 12 Mathematics Half Yearly 2014

Outcomes Addressed in this Question:

H7 Uses the features of a graph to deduce information about the derivative.

SAMPLE SOLUTION

a)

$y = ax^3 + bx - 3$

Through $(-1,-4)$ $\quad -4 = a(-1)^3 + b(-1) - 3$

$-4 = -a - b - 3$

$a + b = 1 \quad -(1)$

$y' = 3ax^2 + b$

Stat Pt $(-1,-4)$ $\quad 0 = 3a(-1)^2 + b$

$0 = 3a + b$

$3a + b = 0 \quad -(2)$

$(2)-(1) \quad 2a = -1$

$a = -\frac{1}{2}$

in $(1) \quad b = \frac{3}{2}$

3 marks – correct values for a and b obtained with a correct method of solving the equations
2 marks – substantial progress towards correct solution
1 mark – one correct equation established

Many students made basic errors in their solutions, including errors in working with negative numbers and errors in solving simultaneous equations.

b) (i)

$V = x^2h \qquad \therefore 32 = x^2h \qquad \rightarrow h = \frac{32}{x^2}$

$A = x^2 + 4xh$

$= x^2 + 4x.\frac{32}{x^2}$

$= x^2 + \frac{128}{x}$

1 mark – correct answer clearly showing how the given equation was reached.

This part was done well by most students.

b) (ii)

$A = x^2 + \frac{128}{x}$

$A' = 2x - \frac{128}{x^2}$

$A'' = 2 + \frac{256}{x^3}$

StatPt $A' = 0 \qquad 2x - \frac{128}{x^2} = 0$

$2x^3 = 128$

$x^3 = 64$

$x = 4$

Test $\qquad A''(4) = 2 + \frac{256}{4^3} = 6 \quad > 0 \;\therefore$ *Minimum*

When $x = 4, \quad h = \frac{32}{4^2} = 2$

$\therefore$ *Dimensions* $4cm \times 4cm \times 2cm$

3 marks – correct dimensions obtained with minimum area being confirmed
2 marks – substantial progress towards correct solution
1 mark – using a correct method to establish that x=4

Students made a number of errors in this question. There were difficulties in differentiating using the quotient rule – it was much simpler to differentiate the two terms separately in this question. Many students did not test to check that this was a minimum, and many students did not give the dimensions of the box, just giving the value for x or unnecessarily calculating the surface area.

c) (i)

$x < 0$ *and* $x > 2$

1 mark – correct answer

This question was well done by most students

c) (ii)

as $x \rightarrow \infty, \quad f'(x) \rightarrow \infty$

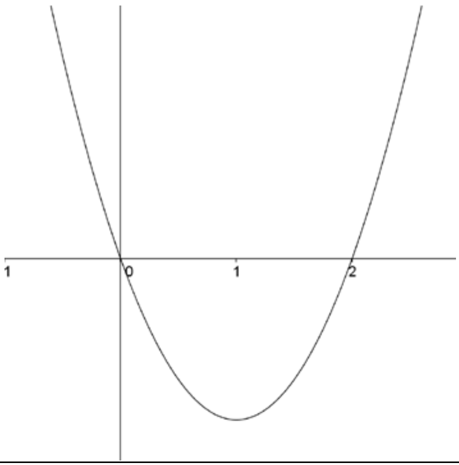
or

f'(x) increases as x increases

1 mark – correct answer

Some students complicated their responses by giving too much unnecessary information.

c) (iii)



2 marks – correctly shaped graph showing intercepts at 0 and 2 and clearly showing the minimum point at x=1.
1 marks – substantial progress towards the correct graph

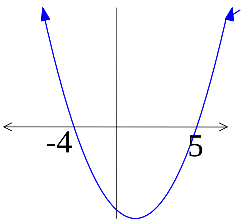
The best graphs were drawn with a template, using a pencil and a ruler to draw the axes, measuring the positions of 0, 1 and 2 and clearly showing the minimum value occurring at x=1.

Year 12 Half Yearly Question No. 9	Mathematics Solutions and Marking Guidelines	Examination 2014

Outcomes Addressed in this Question

P4 Chooses and applies appropriate algebraic and graphical techniques

P5 Understands the concept of a function and the relationship between a function and its graph

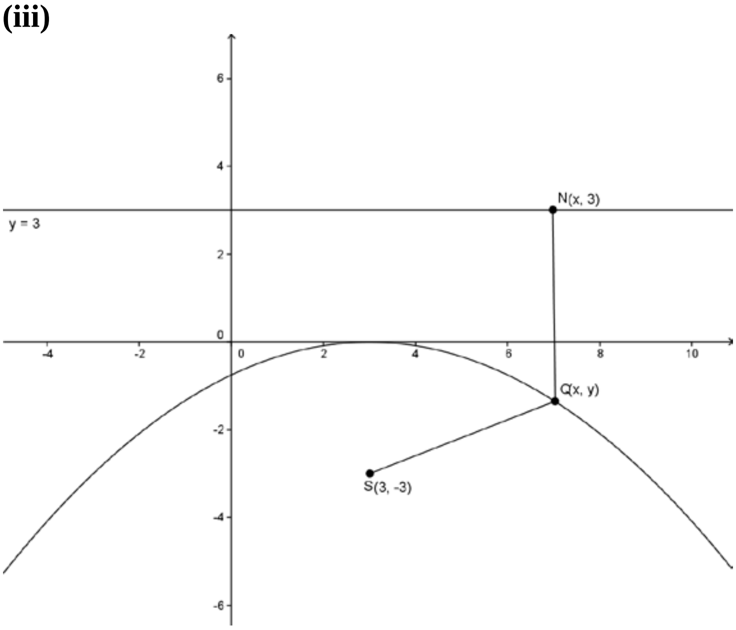
Outcome	Solutions	Marking Guidelines
P 5	(a) For $2x^2-3x+4=0,\ a=2,\ b=-3,\ c=4.$ $\alpha\beta=\frac{c}{a}=\frac{4}{2}=2. \qquad \alpha+\beta=\frac{-b}{a}=\frac{3}{2}.$ $\therefore \alpha+\beta+\alpha\beta=2+\frac{3}{2}=3\frac{1}{2}.$	1 mark : correct answer
	(b) $x^2+1+\frac{25}{x^2+1}=10$ Let $y=x^2+1$, then $y+\frac{25}{y}=10$ $y^2-10y+25=0$ $(y-5)^2=0 \quad \therefore y=5$, and so $x^2+1=5$ $\therefore x^2=4$ and $x=\pm 2.$	2 marks : correct solution 1 mark : substantial progress toward correct solution
P 4	(c) (i) Length of the square l = hypotenuse of a right triangle with shorter sides x and $1-x$. Area of shaded square $A=l^2=x^2+(1-x)^2$ (Pythagoras) $\therefore A=x^2+1-2x+x^2$ $\therefore A=2x^2-2x+1.$	1 mark : correct solution
P 4	(ii) A is a quadratic , whose minimum value occurs on the axis of symmetry. It is a minimum as the co-efficient of x^2 is positive. $x=\frac{-b}{2a}=\frac{2}{4}=\frac{1}{2}.$ When $x=\frac{1}{2},\ A=\frac{1}{2}-1+1=\frac{1}{2}.$ $\therefore$ minimum value of A is $\frac{1}{2}.$	2 marks : correct answer 1 marks : substantial progress toward correct answer
P 4	(d) $20x-17\equiv a(x-4)-b(5x+1).$ $\equiv ax-4a-5bx-b$ $\equiv(a-5b)x+(-4a-b)$ Equating like coefficients, $20=a-5b \quad [1]$ $-17=-4a-b \quad [2]$ $\therefore 80=4a-20b \quad 4\times[1]$ $\therefore 63=-21b$ $\therefore b=-3$ and substituting in $[1]$, $a=5.$	2 marks : correct solution 1 mark : substantial progress toward correct solution
P 4	(e) $x^2+2kx+k+20=0$ has real roots when $\Delta\geq 0.$ $\Delta=b^2-4ac=(2k)^2-4.1.(k+20)$ Solving $4k^2-4k-80\geq 0,$ $(k+4)(k-5)\geq 0,$ From the graph $k\leq -4$ and $k\geq 5.$ 	3 marks : correct answer 2 marks : substantial progress toward correct solution 1 mark : some progress toward correct solution

Year 12 Mathematics Half Yearly Examination 2014

Question No. 10 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

Outcome	Solutions	Marking Guidelines
H5	**(a)** Parabola is of the form: $x^2 = 4ay$ passing through $(6, 2)$ with vertex at the origin $\therefore 36 = 8a$ $a = \frac{9}{2}$ Focus is then $S\left(0, \frac{9}{2}\right)$	**2 marks** Correct solution stating both the focal length and co-ordinates of focus. **1 mark** Substantial progress towards correct solution
H5	**(b) (i)** $(x-3)^2 = -12y$ Here, $a = 3$, vertex $(3,0)$, concave down $\therefore$ Focus is $(3,-3)$	**1 mark** Correct answer
H5	**(ii)** Directrix: y = 3	**1 mark** Correct answer
H5	**(iii)** 	**1 mark** Graph sketched correctly showing important features. Some regard must have been given to scale and positioning of focus and y intercept.
H5	**(iv)** $QS = \sqrt{(x-3)^2 + (y+3)^2}$ $= \sqrt{-12y + (y+3)^2}$ $= \sqrt{-12y + y^2 + 6y + 9}$ $= \sqrt{y^2 - 6y + 9}$ $= \sqrt{(y-3)^2}$ $= y - 3$ $QN = \sqrt{(x-x)^2 + (y-3)^2}$ $= \sqrt{(y-3)^2}$ $= y - 3$ $= QS$ $\therefore$ Q is equidistant from the focus and directrix Alternatively, assume $QS = QN$ and show equation of locus is $(x-3)^2 = -12y$	**2 marks** Correct solution showing $QS = QN$ or alternative solution. **1 mark** Substantial progress towards correct solution.

H5	**(c) (i)** $x^2 = 12y$ Focus $(0, 3)$ Sub. in $5x + 2y = 6$ $0 + 6 = 6$ True $\therefore\ 5x + 2y = 6$ is a focal chord.	**2 marks** Correct solution stating co-ordinates of focus and showing correct substitution. **1 mark** Substantial progress towards correct solution.
H5	**(ii)** If $ax + by = 6$ is a focal chord it passes through $(0,3)$ ie. $a.0 + b.3 = 6$ $b = 2$ $\therefore\ a$ can be any value, and $b = 2$ for line to be a focal chord.	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution.

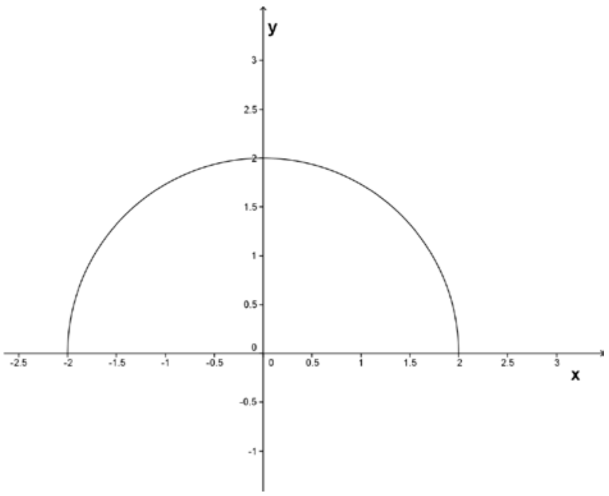
Year 12	Mathematics -Advanced Solutions and Marking Guidelines	Task 2 Half Yearly 2014

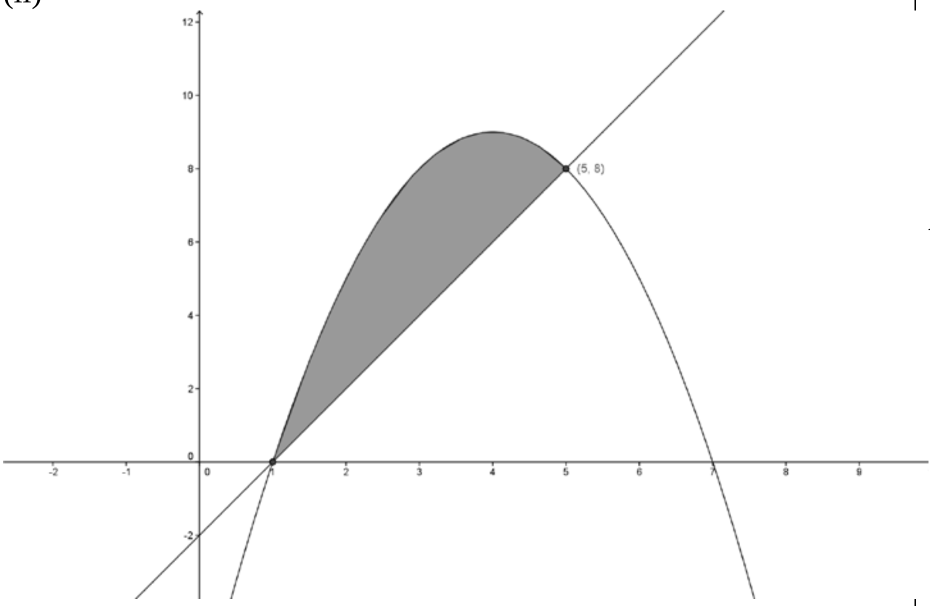
Multiple Choice: 1.A 2C 3A 4B 5C 6D

Question 11

Outcome Addressed in this Question

H8 Uses techniques of integration to calculate areas and volumes.

	Solutions	**Marking Guidelines**
H8	**(a)** (i) $f(-x) = (-x)^3 + (-x)$ $= -x^3 - x$ $= -(x^3 + x)$ $\therefore f(-x) = -f(x)$ So $f(x)$ is an odd function	(a) (i) **1 mark:** correct answer
H8	(ii) Since the function is odd, $\int_{-a}^{a} f(x)dx = 0$ $\therefore \int_{-2}^{2} x^3 + x\,dx = 0$	(ii) **1 mark:** correct answer
H8	**(b)** (i) 	(b) (i) **1 mark:** correct answer *(Diagrams should include intersections on axes, and the function should be a neat semi-circle.)*
H8	(ii) Integral will have the same value as the quarter circle, radius 2 units. $\therefore \int_{0}^{2} \sqrt{4 - x^2}\,dx = \pi$	(ii) **1 mark:** correct answer
H8	**(c)** (i) $2x - 2 = 8x - 7x - x^2$ $x^2 - 6x + 5 = 0$ $(x-5)(x-1) = 0$ $x = 1, 5$ When $x = 1$, $y = 2(1) - 2 = 0$ Hence (1, 0) When $x = 5$, $y = 2(5) - 2 = 8$ Hence (5, 8)	(c) (i) **2 marks:** complete correct solution, including substitution. **1 mark:** Partially correct solution. *("Show that" questions require full working)*

H8	(ii)  	(ii) **2 marks:** Correct shapes of curves and all points of intersection. **1 mark:** Partially correct. *(Finding the x-intercepts of the parabola enable a much neater sketch of the situation.)*
H8	(iii) $A=\int_1^5 8x-7-x^2-(2x-2)dx$ $=\int_1^5 6x-5-x^2\,dx$ $=\left[3x^2-5x-\frac{x^3}{3}\right]_1^5$ $=10\frac{2}{3}\ units^2$	(iii) **3 marks: C**omplete correct solution **2 marks:** Significant progress. **1 mark:** Limited progress.

Outcome Addressed in this Question

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

	Solutions	Marking Guidelines
H5	12. a) $\sum_{k=1}^{4}(-1)^k k^2 = (-1)^1.(1)^2 + (-1)^2.(2)^2 + (-1)^3.(3)^2 + (-1)^4.(4)^2$ $= -1 + 4 - 9 + 16$ $= 10$	**1 mark for correct answer**
H5	b) i). The series is 750, 850, … with $a = 750$, $d = 100$ $T_n = a + (n-1)d$ $= 750 + (n-1) \times 100$ $= 750 + 100n - 100$ $= 100n + 650$	**1 mark for correct answer**
H5	ii). Use $n = 10$, $T_n = 100n + 650$ $T_{10} = 100 \times 10 + 650$ $= 1650$ $\therefore$ she swims 1650m on 10th day	**1 mark for correct answer**
H5	iii). Using $S_n = \frac{n}{2}[2a + (n-1)d]$ with $a = 750$, $d = 100$ and $n = 10$ $S_{10} = \frac{10}{2}[2(750) + (10-1)100]$ $= 5(1500 + 900)$ $= 12\ 000$ $\therefore$ distance is 12 000m or 12 km	**1 mark for correct answer**
H5	iv). Using $S_n = \frac{n}{2}[2a + (n-1)d]$ with $a = 750$, $d = 100$ and $S_n = 34\ 000$ $34\ 000 = \frac{n}{2}[2(750) + (n-1)100]$ $68\ 000 = n[1500 + 100n - 100]$ $68\ 000 = n[1400 + 100n]$ $100n^2 + 1400n - 68\ 000 = 0$ $n^2 + 14n - 680 = 0$ $(n + 34)(n - 20) = 0$ $n = -34, 20$ As $n > 0$, then $n = 20$ $\therefore$ 20 days	**2 marks for complete correct solution** **1 mark for correctly substituting into formula**

H5	c) i). As 7.2% p.a. = 0.006 per month $A_n = 200\,000r^n - M(1 + r + r^2 + \cdots + r^{n-1})$, $A_{300} = 200\,000 \times 1.006^{300} - M(1 + 1.006 + 1.006^2 + \cdots + 1.006^{299})$ $0 = 200\,000 \times 1.006^{300} - M(1 + 1.006 + 1.006^2 + \cdots + 1.006^{299})$ $\therefore M = 200\,000 \times 1.006^{300} \div (1 + 1.006 + \ldots + 1.006^{299})$ Now, for $1 + 1.006 + \ldots + 1.006^{299}$, $a = 1$, $r = 1.006$ and $n = 300$ and $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_n = \frac{1(1.006^{300} - 1)}{1.006 - 1}$ $\therefore M = 200\,000 \times 1.006^{300} \div \frac{1(1.006^{300} - 1)}{1.006 - 1}$ $= 1439.177383$ $= 1439.18$ (2 dec pl) $\therefore$ the repayment is \$1439.18	**3 marks for complete correct solution** **2 marks for correct equation for M** **1 mark for correct equation for A_{300}**
H5	ii). Use $M = 200\,000 \times 1.006^n \div \frac{1(1.006^n - 1)}{1.006 - 1}$ and $M = 2800$ $2800 = 200\,000 \times 1.006^n \times \frac{0.006}{1.006^n - 1}$ $2800(1.006^n - 1) = 1200 \times 1.006^n$ $2800 \times 1.006^n - 2800 = 1200 \times 1.006^n$ $1600 \times 1.006^n = 2800$ $1.006^n = \frac{2800}{1600}$ $1.006^n = 1.75$ $\log 1.006^n = \log 1.75$ $n \log 1.006 = \log 1.75$ $n = \frac{\log 1.75}{\log 1.006}$ $= 93.54882691$ $\therefore$ repaid after 94 months	**2 marks for complete correct solution** **1 mark for correctly substituting 2800 into equation for M**